5.0 INTRODUCTION

This chapter is about the operating principles, types and applications of a conventional transformer and auto transformer. It also explains about the relationship between primary and secondary in terms of winding, current and voltage of transformers and power losses in transformer. The learning outcomes for this topic is student should be able to identify the operations of transformers based on the principles of electromagnet.

5.1 TRANSFORMERS

A transformer is a device which uses the phenomenon of mutual induction to change the values of alternating voltages and currents. In fact, one of the main advantages of alternating current transmission and distribution is the ease with which an alternating voltage can be increased or decreased by transformers.

Losses in transformers are generally low and thus efficiency is high. Being static they have a long life and are very stable. Transformers range in size from the miniature units used in electronic applications to the large power transformers used in power stations.

5.1.1 Transformer Construction

A transformer consists of two windings connected by a magnetic core. Primary winding is connected to a power supply and the other winding is connected to a load. A diagram for a transformer construction is shown in Figure 5.1.
5.1.2 Operating Principles

When an alternating voltage, $V_{AC}$ is applied to a primary winding and the secondary winding is an open-circuit, a small current will flow in the primary winding which produces induced emf, $e_1$ in the laminated core. This alternating flux will link with secondary coils then produces mutual induced emf of $e_2$. 

Transformers
The induced emf, $E$ in a coil of $N$ turns is given by

$$E = -N \frac{d\Phi}{dt} \text{ volts} \quad [5.1]$$

where $\frac{d\Phi}{dt}$ is the rate of change of flux.

emf generates due to changes of magnetic flux, $e_1$ as equation 2.1 in Chapter 2.

$$e_1 = -N \frac{d\phi}{dt}$$

emf generates due to changes of current, is as equation 2.2 in Chapter 2.

$$e_2 = -L \frac{di}{dt}$$

Refer to equation 2.3, Faraday’s Law

$$e_1 = e_2$$

$$-N \frac{d\phi}{dt} = -L \frac{di}{dt}$$

$$L = N \frac{d\phi}{dt} \cdot \frac{dt}{di}$$

$$L = N \frac{d\phi}{di}$$

In the ideal transformer, the rate of change of flux is the same for both primary and secondary

$$\frac{E_1}{N_P} = \frac{E_2}{N_S} \quad [5.2]$$

The induced emf per turn is constant. By assuming no losses, $E_1 = V_P$ and $E_2 = V_S$

$$\frac{V_P}{N_P} = \frac{V_S}{N_S} \quad \text{or} \quad \frac{V_P}{V_S} = \frac{N_P}{N_S} \quad [5.3]$$
When load is connect across the secondary winding, a current $I_s$ flows. In an ideal transformer, losses are neglected and a transformer is considered to be 100% efficient.

Hence input powers = output power or $V_P I_P = V_S I_S$, the primary and secondary volt-amperes are equal.

Thus

$$\frac{V_P}{V_S} = \frac{I_S}{I_P} \quad [5.4]$$

Combining equations 5.3 and 5.4 gives the relationship of primary and secondary in terms of winding, voltage and current.

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} = \frac{I_S}{I_P} \quad [5.5]$$

### 5.1.3 Transformer Ratio

Transformer ratio is the comparison between primary and secondary elements. Flux which produced by primary winding completely inducing to secondary winding. Therefore, emf induced in all windings are similar for primary and secondary winding.

$$\frac{\text{Total of emf induced in secondary winding}}{\text{Total of emf induced in primary winding}} = \frac{N_s \times \text{emf / winding}}{N_p \times \text{emf / winding}} = \frac{N_p}{N_s} \quad [5.6]$$

Constant $K$ is known as voltage transformation ratio, then

$$K = \frac{N_s}{N_p} = \frac{E_2}{E_1} = \frac{V_s}{V_p} \quad [5.7]$$

If $N_s < N_p$ then $K < 1$ therefore this transformer is called step-down transformer. If $N_s > N_p$ then $K > 1$ therefore this called step-up transformer. For $N_s = N_p$ then $K = 1$ therefore this is coupling transformer.
Example 5.1

A transformer is to be used to provide a 60 V output from a 240 V AC supply. Calculate:
   i) the turns of ratio required and
   ii) the number of primary turns, if the secondary is wound with 500 turns.

Solution 5.1

\[ V_2 = 60 \text{ V}, \quad V_1 = 240 \text{ V} \text{ and } N_2 = 500 \]

i) \[ K = \frac{V_2}{V_1} = \frac{60}{240} \text{, So turns ratio, } \frac{1}{4} \text{ or } 1:4 \]

ii) \[ \frac{N_1}{500} = \frac{4}{1} \text{, Therefore, } N_1 = 2000 \]

Example 5.2

A 2000/200V, 20kVA transformer has 66 turns in the secondary. Calculate:
   i) primary turns
   ii) primary and secondary full-load currents

Neglect the losses.
   i) Smooth wave ripples in the DC circuit.
   ii) Improve the transmission characteristics of waves in the telephone line

Solution 5.2
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i) \( \frac{N_2}{N_1} = K = \frac{1}{10} \)

\( N_1 = N_2 \times 10 = 66 \times 10 = 660 \text{ turns} \)

ii) \( V_1I_1 = V_2I_2 = 20 \times 10^3 \)

\( I_2 = \frac{20 \times 10^3}{200} = 100 \text{A} \)

Example 5.3

The numbers of windings for the three transformers are as below:

a) \( N_p = 100, N_s = 2000 \)

b) \( N_p = 3000, N_s = 2000 \)

c) \( N_p = 100, N_s = 100 \)

Calculate the value of \( K \) for each transformer then determine the type of transformer and draw the symbol of transformer to differentiate the number of windings

Solution 5.3

a) \( K = \frac{N_s}{N_p} = \frac{2000}{100} = 20 \)

\( \therefore K > 1 \implies \text{step-up transformer} \)

b) \( K = \frac{N_s}{N_p} = \frac{2000}{3000} = 0.67 \)

\( \therefore K < 1 \implies \text{step-down transformer} \)

c) \( K = \frac{N_s}{N_p} = \frac{100}{100} = 1 \)

\( \therefore K = 1 \implies \text{coupling transformer} \)
5.2 TRANSFORMER TYPES

There are two types of transformer known as core types and shell type. The core type transformer is where the winding surround considerable part of the core and shell type is where the core surround is considerable portion of the winding.

![Core Type and Shell Type](image)

**Figure 5.3: (a) Core Type (b) Shell Type**

Single phase core type transformer consists of two windings and one core. Meanwhile, shell type consists of one winding and two cores. Shell types provide a higher degree of mechanical protection to the winding because it is surrounded by core. On the other hand, core type provides better insulation between the primary and secondary because of its inherent two winding. Shell type transformer is used in transmission of multiple high and low voltages. The main application for a core type transformer is to increase voltage from a standard 240v supply up to 415v, instead of using three phase supply. The advantages of each type are:

i. Core type is the most commonly used method of construction, the smaller core means less weight and expense.

ii. Shell type is used for larger transformer because they can be made with a reduce height.

5.3 TRANSFORMER EMF EQUATION

![Waveform of Flux Variation](image)

**Figure 5.4: Waveform of Flux Variation**
The maximum value of flux to be $\Phi_m$, webers and the frequency to be $f$, hertz. The flux has to change from $+\Phi_m$ to $-\Phi_m$ in the half cycle, namely in $\frac{1}{2f}$ seconds.

Average rate of change of flux = $2 \Phi_m \div \frac{1}{2f}$

$$= 4f \Phi_m \text{ webers per second}$$

and average emf induced per turn is

$$= 4f \Phi_m \text{ volts} \quad [5.8]$$

For a sinusoidal wave the rms or effective value is 1.11 times the average value, therefore the rms value of emf induced per turn is as shown in equation 5.9

$$E = 1.11 \times 4f \Phi_m \quad [5.9]$$

Hence the rms value of emf induced in primary is

$$E_1 = 4.44 N_P f \Phi_m \text{ volts} \quad [5.10]$$

and rms value of emf induced in secondary is

$$E_2 = 4.44 N_S f \Phi_m \text{ volts} \quad [5.11]$$

**Example 5.4**

A 250 kVA, 1100 V / 400 V, 50 Hz single-phase transformer has 80 turns on a secondary. Calculate:

a) the approximate values of the primary and secondary currents.

b) the approximate number of primary turns.

c) the maximum values of flux.

**Solution 5.4**

a) Full – load primary current

$$I_p = \frac{250 \times 1000}{1100} = 22.7 \text{ A}$$
Full – loaded secondary current, \( I_S = \frac{250 \times 1000}{400} = 625 \text{ A} \)

b) Number of primary turns, \( N_P = \frac{80 \times 11000}{400} = 2200 \text{ turns} \)

c) Maximum value of flux, \( E_2 = 4.44 N_2 f \Phi_m \text{ volts} \)
\[
400 = 4.44 \times 80 \times 50 \times \Phi_m
\]
\( \Phi_m = 22.5 \text{ mWb} \)

Example 5.5

An ideal 25 kVA transformer has 500 turns on the primary winding and 40 turns on the secondary winding. The primary is connected to 3000 V, 50 Hz supply. Calculate
a) primary and secondary currents on full-load
b) secondary emf and
c) the maximum core flux

Solution 5.5

a) \( I_p = \frac{S}{V_p} = \frac{25 \times 10^3}{3000} = 8.33 \text{ A} \)

\[
I_S = \frac{I_p N_S}{N_P} = \left[\frac{8.33}{500}\right] \times 40 = 0.67 \text{ A}
\]

b) \( E_2 = \frac{E_1 N_S}{N_P} = \left[\frac{3000}{500}\right] \times 40 = 240V \)

\( E_1 = 4.44 f N_p \phi_m \)

c) \( \phi_m = \frac{3000}{4.44 \times 50 \times 500} = 27 \times 10^{-3} \text{ wb} = 27 \text{ mWb} \)
5.4 AUTO TRANSFORMER

An autotransformer has only a single winding which is tapped at some point along the winding. The primary and secondary windings are not electrically separate, hence if an open-circuit occurs in the secondary winding the full primary voltage appears across the secondary.

![Diagram of Autotransformer](image)

**Figure 5.5: An Auto Transformer (a) Step Down (b) Step Up**

AC voltage is applied across a portion of the winding, a lower and higher voltage is produced across another portion of the same winding for step down and step up auto transformer. An auto transformer will produce a lower voltage when connected from one end to the tap, and the higher voltage when connected to the end of the ends of the winding.

By neglecting the losses, the leakage reactance and the magnetizing current, so the ratio, \( n \) of the smaller voltage to the larger voltage is as in equation 5.12 below

\[
 n = \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} \tag{5.12}
\]

Generally, an auto transformer should not be used for interconnecting high voltage and low voltage systems. The most important thing is the common connection to the earth, otherwise there is a risk of serious shock. An auto transformer which the voltage ratios not exceeding about 3:1, it is smaller, lighter, cheaper and more efficient than an isolating (two winding) transformer of the same rating. Large three phase auto transformers are used in electric power distribution systems to interconnect 33 kV and 66 kV sub transmission networks.

The nearer the ratio of transformation is to unity, the greater is the economy of conductor material. Also, for the same current density in the windings and the same
peak values of the flux and of the flux density, the $I^2R$ loss in the auto transformer is lower and the efficiency higher than in the two winding transformer.

Auto transformer are mainly used for interconnecting systems that are operating at roughly the same voltage and starting cage-type induction motors. The advantages of auto transformers over double wound transformers included:

i. saving in a cost since less copper is needed.
ii. less volume, hence less weight.
iii. higher efficiency, resulting from lower $I^2R$ losses
iv. continuously variable output voltage is achievable if a sliding contact is used.
v. smaller percentage voltage regulation.

The disadvantages of the auto transformer are that:

i. There is a direct metallic connection between the input and the output, where the coupling in a double wound transformer is magnetic only when giving electrical isolation of the two winding.

ii. In the event of an open circuit fault in the common part of the winding, the input voltage of a step down autotransformer would appear on the output terminals.

### 5.4 Transformer Losses and Efficiency

An ideal transformer is one which has no losses, its winding have no ohmic resistance, there is no magnetic leakage and hence no $I^2R$ and core losses. In other words, an ideal transformer consists of two purely inductive coils wound on the loss free core.

There are two cases consider for transformer with losses but no magnetic leakage.

i) No Load

ii) Loaded/ Full load

No load transformer is assumed as an ideal transformer which there is no losses. The losses only can occur in a loaded transformer. There are two types of losses on load transformer which is iron loss in the core and copper loss in the winding.

a) Copper losses are variable and result in a heating of the conductors, due to the fact that they possess resistance. If $R_1$ and $R_2$ are the primary and secondary winding resistances than the total copper loss is as shown in equation 5.13 below

\[
\text{Copper loss} = I_1^2R_1 + I_2^2R_2 \tag{5.13}
\]
b) Core losses due to hysteresis and eddy currents.

i) Hysteresis loss is the heating of the core as a result of the internal molecular structure reversals which occur as the magnetic flux alternates. The loss is proportional to the area of the hysteresis loop and thus low loss nickel iron alloys are used for the core since their hysteresis loop have small areas.

ii) Eddy current loss is the heating of the core due to emf's being induced not only in the transformer windings but also in the core. The induced emf's set up circulating currents call ed eddy currents. Owing to the low resistance of the core, eddy currents can be quite considerable and can cause a large power loss and excessive heating of the core.

Since the maximum value of the flux in a normal transformer does not vary by more than about 2 per cent between no load and full load, it is usual to assume the core loss constant at all loads.

Hence, if $P_c =$ total core loss,

Then the total losses in transformer are

$$P_c + I_1^2R_1 + I_2^2R_2$$

[5.14]

Efficiency is as well as providing for the output power, into to a transformer must supply the transformer losses. Thus:

$$\text{Input Power} = \text{Output Power} + \text{Power Losses}$$

[5.15]

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} \times 100\%$$

[5.16]

$$= \frac{\text{input power - power losses}}{\text{input power}} \times 100\%$$

[5.17]

Or

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power} + \text{losses}} \times 100\%$$

[5.18]

$$\text{Efficiency} = \frac{I_2V_2 \times \text{p.f.}}{I_2V_2 \times \text{p.f.} + P_c + I_1^2R_1 + I_2^2R_2}$$

[5.19]
Greater accuracy is possible by expressing the efficiency thus:

\[ \eta = 1 - \frac{\text{losses}}{\text{input power}} \quad [5.20] \]

**Example 5.6**

The primary and secondary windings of a 500 kVA transformer have resistances of 0.42 Ω and 0.0019 Ω respectively. The primary and secondary voltages are 11 000 V and 400 V respectively and the core loss is 2.9 kW, assuming the power factor of the load to be 0.8. Calculate the efficiency on:

(a) full load
(b) half load

**Solution 5.6**

(a) Full load:

Primary current, \( I_1 \) = \( \frac{S}{V_1} \)  
\[ = \frac{500 \times 1000}{11000} = 45.5 \text{ A} \]

Secondary current, \( I_2 \) = \( \frac{S}{V_2} \)  
\[ = \frac{500000}{400} = 1250 \text{ A} \]

Total losses = \( P_c + I_1^2R_1 + I_2^2R_2 \)  
\[ = 2.9 \text{ k} + (45.5)^2 \times 0.42 + (1250)^2 \times 0.0019 \]
\[ = 2.9 \text{ k} + 870 + 2969 \]
\[ = 6.74 \text{ kW} \]

Output power = 500 k × 0.8 = 400 kW

Input power = Output power + losses = 400 k + 6.74 k = 406.74 kW
\[ \text{Efficiency, } \eta = 1 - \frac{\text{losses}}{\text{input power}} \]

\[ \eta = \left(1 - \frac{6.74}{406.74}\right) = 0.983 \text{ per unit} \]

\[ \eta = 98.3\% \]

(b) Half Load:

Since losses varies as the square of current

\[ \text{Total losses} = P_c + \left(I_1/2\right)^2 R_1 + \left(I_2/2\right)^2 R_2 \]

\[ = 2.9 \text{ k} + \left(45.5/2\right)^2 \times 0.42 + \left(1250/2\right)^2 \times 0.0019 \]

\[ = 2.9 \text{ k} + 217 + 742 \]

\[ = 3.85 \text{ kW} \]

Input power \[ = \frac{406.74 \text{ k}}{2} \]

\[ = 203.37 \text{ kW} \]

\[ \therefore \text{Efficiency} = \left(1 - \frac{3.85}{203.37}\right) \]

\[ = 0.981 \text{ per unit} \]

\[ = 98.1\% \]

Example 5.7

In a 50 kVA transformer, the iron loss is 500 W and full-load copper loss is 800W. Find the efficiency at full-load and half-load at 0.8 p.f. lagging.

Solution 5.7

Full-load, 0.8 p.f

Output Power \[ = 50 \text{ k} \times 0.8 = 40 \text{ kW} \]

Total Losses \[ = \text{iron loss + copper loss} \]

\[ = 500 + 800 \]

\[ = 1300 \text{ W} \]

\[ = 1.3 \text{ kW} \]
Input Power  
= Output power + Losses
= 40 k + 1.3 k
= 41.3 kW

Efficiency, $\eta$  
= \frac{\text{output power}}{\text{input power}} \times 100
= \frac{40}{41.3} \times 100
= 96.85 \%

Half-load, 0.8 p.f

Output Power  
= (50 k /2) \times 0.8 = 20 kW

Total Losses  
= 500 + (1/2)^2 \times 800 = 700 W = 0.7 kW

Input Power  
= 20 k + 0.7 k = 20.7 kW

Efficiency, $\eta$  
= \frac{20}{20.7} \times 100 = 96.6 \%

REFERENCE


PROBLEMS

1. A transformer has 500 primary turns and 3000 secondary turns. If the primary voltage is 240 V, determine the secondary voltage, assuming an ideal transformer.  
   (1440 V)

2. An ideal transformer with a turns ratio of 2 : 7 is fed from 240 V supply. Determine its output voltage.  
   (840 V)

3. An ideal transformer has a turns ratio of 8 : 1 and the primary current is 3 A when it is supplied at 240 V. Calculate the secondary voltage and current.  
   ($V_2 = 30$ volts, $I_2 = 24$ A)
4. An ideal transformer connected to a 240 V mains, supplies a 12 V, 150 W lamp. Calculate the transformer turns ratio and the current taken from the supply.
   \( \text{turn ratio} = 20, I_1 = 0.625 \text{ A} \)

5. How many turns are required for a 36 V secondary if a 240 V primary has 600 N.
   (90 turns)

6. If the primary winding of a 1:2 step-up transformer has 3 N/V, the secondary winding will have 6 N/V. True or false?
   (False)

7. True or false? The secondary flux of a transformer cancels part of the primary flux.
   (True)

8. A 100 kVA, 4000 V/200 V, 50 Hz single phase transformer has 100 secondary turns. Determine:
   a. the primary and secondary current
   b. the number of primary turns
   c. the maximum value of the flux
   \( I_1 = 25 \text{ A}; I_2 = 500 \text{ A}; N_1 = 2000 \text{ turns}; \Phi_m = 9.01 \text{ mWb} \)

9. A 4500 V/225 V, 50 Hz single phase transformer is to have an approximate e.m.f per turns of 15 V and operate with a maximum flux of 1.4 T. Calculate:
   (a) the number of primary and secondary turns
   (b) the cross sectional area of the core
   \( N_1=300, N_2= 15 \)

10 A single phase 2200/250V, 50 Hz transformer has a net core area of 36cm\(^2\) and a maximum flux density of 6 wb/m\(^2\). Calculate the number of primary and secondary turns.
    \( A = 0.0483 \text{ m}^2; N_1 = 459, N_2 = 52 \)

11 A 50kVA, 6600/250V transformer has 52 secondary turns. Find
   (i) the number of primary turns
   (ii) full-load primary and secondary currents.
      (Neglect losses)
      \( 1373 \text{ turns}, 7.58 \text{ A}, 200 \text{ A} \)

12 The net cross-sectional area of the core of 400/3000V, 50Hz transformer is 600cm\(^2\). If the maximum flux density in the core is 1.3 wb/m\(^2\), find the number of primary and secondary turns.
    \( 24 \text{ turns}; 198 \text{ turns} \)

13 A 60 kVA, 1600 V/100 V, 50 Hz, single phase transformer has 50 secondary windings. Calculate:
   (i) the primary and secondary current
   (ii) the number of primary turns
(iii) the maximum value of the flux

(37.5 A, 600 A, 800 turns, 9.0 mWb)

14 A single phase, 50 Hz transformer has 40 primary turns and 520 secondary turns. The cross sectional area of the core is 270 cm$^2$. When the primary winding is connected to a 300 V supply, determine:
(i) the maximum value of flux density in the core
(ii) the voltage induced in the secondary winding

(1.25 T, 3.90 kV)

15 A 3.3 kV/110 V, 50 Hz, single phase transformer is to have an approximate emf per turn of 22 V and operate with a maximum flux of 1.25 T. Calculate
(i) the number of primary and secondary turns
(ii) the cross sectional area of the core

(150 turns, 5 turns, 792.8 cm$^2$)

16 200 kVA rated transformer has a full loaded copper loss of 1.5 kW and an iron loss of 1 kW. Determine the transformer efficiency at full load and 0.85 power factor.

(0.9855 per unit or 98.55 %)

17 400 kVA transformer has a primary winding resistance of 0.5 Ω and a secondary winding resistance of 0.001Ω. The iron loss is 2.5 kW and the primary and secondary voltages are 5 kV and 320 V respectively. If the power factor of the load is 0.85, determine the efficiency of the transformer:

i. on a full load
ii. on half load

(97.91 %, 97.87 %)

18 A single phase transformer has a voltage ratio of 6:1 and the high voltage winding is supplied with 540 V. The secondary winding provides a full load current of 30 with power factor of 0.8 lagging. Neglecting losses, find:

i. a rating of the transformer
ii. the power supplied to load
iii. the primary current.

(2.7 kVA, 2.16 kW, 5 A)